

On a Collinear System for the Emission and Reception of Standing Gravitational Waves

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We introduce an experimental concept, based on the Gertsenshtein-Zeldovich effect, for the generation and detection of standing gravitational waves (SGWs) from TEM stationary waves propagating into an external transverse static magnetic field. Although the amplitude of the generated SGWs is extremely faint, they could be detected through the energy loss in the generator and the electromagnetic fields remotely induced in a detector, once accumulated over a large period of time. If successful, such experiment would constitute the first control of gravity opening the way to new tests of the equivalence principle in the quantum vacuum.

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Out of the four fundamental interactions, gravitation remains the only one not to be under technological control. This puts limitations on experimental gravity: while we passively explore the permanent *natural* gravitational fields generated by inertial masses, we do not attempt to *artificially* bend spacetime at will in our laboratories. Actually, generating gravitational fields does not belong to science-fiction: it is a natural possibility offered by Einstein's Equivalence Principle. The universality of free fall teaches us that all types of energy, associated to any of the four fundamental forces, undergo an external gravitational field in the same way. But this also implies that all types of energies produce gravity in the same way. Since we cannot switch off the binding energies of matter (inertial) sources, one should rely on electromagnetic (EM) energies as a source of *human-made* controllable gravitational fields. Therefore, controlling gravity in a lab requires not only to generate, but also to detect, such artificial gravitational fields.

The problem of gravity control in those terms has been considered for decades. While Weber [1] envisioned the importance of both the generation and detection of gravitational waves (GWs) as early as 1960, a key mechanism to convert EM fields into gravitational waves has been theoretically discovered by Gertsenshtein [2] in 1962 and applied to astrophysics a decade later by Zeldovich [3]. This mechanism is based on wave resonance: a static magnetic field perpendicular to an incident EM wave produces GWs since the equivalence principle couples both Einstein and Maxwell wave systems. Investigation on electromagnetic detectors of GWs have then been started in [4, 5] before a first design of electromagnetic generator of standing gravitational waves (SGWs) was proposed in [6]. This design does not make use of the Gertsenshtein effect and is constituted of a toroidal resonant EM cavity producing cylindrical SGWs along the axis of symmetry. The design we propose here has a simpler collinear geometry and,

thanks to the coupling with a stronger external magnetic field (Gertsenshtein effect), it is more efficient. Over the years, several other experimental concepts have been proposed and reviewing them is out of the scope of this letter.

The extreme weakness of the conversion process of EM waves into GWs constitutes a serious obstacle to any possible practical realization of the so-called Gertsenshtein effect. Although several proposals [7–9] have tried to overcome this, it seems to us that quite some work is still needed to make this exciting perspective of gravity control an experimental reality.

In this paper, we adopt a different strategy to make practical use of the Gertsenshtein effect through the generation of standing gravitational waves (SGWs) produced from two generators in which standing TEM waves are plunged into a transverse intense magnetic field. These SGWs could be maintained during large period of time to produce detectable effects in spite of their extreme faintness at generation.

Electromagnetic generators of SGWs. The Gertsenshtein effect [2] exploits the coupling between an EM wave and an external static magnetic field to produce GWs. It can be formulated in the linearized approach of Einstein-Maxwell equations, for perturbations of a background Minkowski space:

$$g_{\mu\nu} = \eta_{\mu\nu} + c_{\mu\nu} + w_{\mu\nu} + h_{\mu\nu} \quad (1)$$

where $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ is the Minkowski metric and $c_{\mu\nu}, w_{\mu\nu}, h_{\mu\nu} \ll 1$ represent the metric perturbations respectively due to (i) the external static magnetic field generated by some coil ($c_{\mu\nu}$), (ii) the EM wave ($w_{\mu\nu}$) and (iii) the coupling between the external magnetic field and the EM wave ($h_{\mu\nu}$). Case (i) has been studied in [10], case (ii) in [11, 12] and case (iii) in [2, 3]. The generator of SGWs in [6] actually belongs to the case (ii), since the GWs are purely generated by EM standing waves with no static external magnetic field. When one con-

siders a straight propagation of the EM wave, then only the last perturbation, $h_{\mu\nu}$, contains transverse GWs (see also [11, 12]) and will retain our attention here. In [6], the periodic configuration of the EM standing wave acting as the only source in the toroidal generator allows transverse perturbation modes to be excited.

Assuming Lorenz gauge condition $\partial_\mu h^{\mu\nu} = 0$, the linearized Einstein equations can be written down (in S.I. units):

$$\square^2 h_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}^{(\text{res})} \quad (2)$$

where $T_{\mu\nu}^{(\text{res})}$ is the resonant stress-energy tensor given by

$$T_{\mu\nu}^{(\text{res})} = T_{\mu\nu}^{(\text{tot})} - T_{\mu\nu}^{(\text{c})} - T_{\mu\nu}^{(\text{w})} \quad (3)$$

where $T_{\mu\nu}^{(X)}$ ($X = \text{c, w, tot}$) are the Maxwell stress-energy tensors in Minkowski space for the EM fields of the coil, the wave and their superposition, respectively:

$$T_{\mu\nu}^{(X)} = -\frac{1}{\mu_0} \left(\eta^{\alpha\beta} F_{\mu\alpha}^{(X)} F_{\nu\beta}^{(X)} - \frac{1}{4} \eta_{\mu\nu} F_{\alpha\beta}^{(X)} F^{(\text{X})\alpha\beta} \right) \quad (4)$$

with the total (antisymmetric) Faraday tensor $F_{\mu\nu}^{(\text{tot})} = F_{\mu\nu}^{(\text{c})} + F_{\mu\nu}^{(\text{w})}$ being due to the superposition of the EM fields of the coil and of the wave. $T_{\mu\nu}^{(\text{res})}$ is traceless (from the basic properties of Maxwell stress-energy tensor) and so does $h_{\mu\nu}$ through Eq.(2). Lorenz gauge condition is compatible with Eq.(2) provided the source $T_{\mu\nu}^{(\text{res})}$ verifies $\partial_\mu T_{(\text{res})}^{\mu\nu} = 0$ or, equivalently, that the zeroth order components $F_{\mu\nu}^{(X)}$ of the Faraday tensor are solutions of the Maxwell equations in Minkowski space $\partial_\mu F_{(\text{X})}^{\mu\nu} = 0$. Equations (2-4) describe the direct Gertsenshtein effect: the conversion of an EM wave into a GW in the presence of an external static magnetic field transverse to the direction of the wave propagation.

We will work in cartesian harmonic coordinates $(x^0, x^1, x^2, x^3) = (ct, x, y, z)$ and consider a TEM standing wave arising from the superposition of two progressive waves counter-propagating along the z-direction:

$$\begin{aligned} F_{01}^{(\text{w}), (0)} &= E_x^\leftarrow + E_x^\rightarrow \\ F_{02}^{(\text{w}), (0)} &= E_y^\leftarrow + E_y^\rightarrow \\ F_{13}^{(\text{w}), (0)} &= (-E_x^\leftarrow + E_x^\rightarrow)/c \\ F_{23}^{(\text{w}), (0)} &= (-E_y^\leftarrow + E_y^\rightarrow)/c \end{aligned} \quad (5)$$

This TEM standing wave is plunged into a transverse static magnetic field

$$\begin{aligned} F_{13}^{(\text{c}), (0)} &= B_y \\ F_{23}^{(\text{c}), (0)} &= -B_x. \end{aligned}$$

This gives rise to the following source of the direct Gertsenshtein effect Eq.(2)

$$\begin{aligned} \tau_{00} &= B_y E_x^\leftarrow - B_x E_y^\leftarrow - B_y E_x^\rightarrow + B_x E_y^\rightarrow = \tau_{00}^\leftarrow + \tau_{00}^\rightarrow \\ \tau_{11} &= B_y E_x^\leftarrow + B_x E_y^\leftarrow - B_y E_x^\rightarrow - B_x E_y^\rightarrow = -\tau_{22} \\ \tau_{12} &= -B_x E_x^\leftarrow + B_y E_y^\leftarrow + B_x E_x^\rightarrow - B_y E_y^\rightarrow = \tau_{21} \end{aligned} \quad (6)$$

where $\tau_{33} = \tau_{00}$ and $\tau_{03} = \tau_{00}^\leftarrow - \tau_{00}^\rightarrow$ ($\tau_{\mu\nu} = \mu_0 T_{\mu\nu}^{(\text{res})}/c$). Given the above structure of $T_{\mu\nu}^{(\text{res})}$, the resulting metric perturbations must have the form:

$$h_{\mu\nu} = \begin{pmatrix} h_\leftarrow + h_\rightarrow & 0 & 0 & h_\leftarrow - h_\rightarrow \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ h_\leftarrow - h_\rightarrow & 0 & 0 & h_\leftarrow + h_\rightarrow \end{pmatrix}, \quad (7)$$

so that the transverse GW perturbations $h_{+, \times}$ are produced by this mechanism of wave resonance[15].

In order to provide a simple description of a collinear electromagnetic GW generator, we will work with the plane wave assumption by writing down:

$$T_{\mu\nu} = \frac{E_0 B_0 \mathcal{A}}{c \mu_0} \delta(x) \delta(y) f_{\mu\nu}(z, t) \quad (8)$$

where E_0 , B_0 stand for the amplitude of the TEM wave and the external magnetic field respectively and where \mathcal{A} is the effective section (in m^2) of the generator. Under this assumption, the solution of the 3D wave equation Eq.(2) along the z-axis outside of the generator is given by

$$h_{\mu\nu}(z, t) = -\mathcal{G}_Z \int_{\text{generator}} \frac{f_{\mu\nu}(z', ct - |z - z'|)}{|z - z'|} dz' \quad (9)$$

where

$$\mathcal{G}_Z = \frac{4GB_0 E_0 \mathcal{A}}{c^5 \mu_0} \quad (10)$$

is the *Gertsenshtein-Zeldovich number* (GZ) associated to a given GWs generator (see also [2, 3]). This GZ number measures the efficiency of the given generator as the instantaneous rate of conversion of EM waves into gravitational ones. The amplitude of the metric perturbations $h_{\mu\nu}$ are therefore of order of magnitude \mathcal{G}_Z , up to some geometrical factor given notably by the length of the generator and the distance to it. The characteristic amplitude of metric perturbations in the toroidal SGWs generator of [6] or those associated with travelling EM wave in [11, 12] correspond to $B_0 = E_0/c$ in Eq.(10). Therefore, in a GW generator relying solely on toroidal EM waves (and not on the Gertsenshtein effect), an electric field of about 10^{10} V/m is required to beat the efficiency of the present model of generator with an external magnetic field of 10T.

If we consider the highest currently achievable electric fields of 10^{15} V/m over a surface of 1cm^2 in petawatt

lasers plunged into a transverse (pulsed) magnetic field of 100T, we get that the generator will produce GW of only 10^{-33} in amplitude over very short time scales (typically those of the intense laser). The associated power loss in the generator can be obtained through the energy carried by the fleeing GWs (see for instance [13]) and is of order of magnitude

$$\left| \frac{dE}{dt} \right| \sim \frac{GE_0^2 B_0^2 \mathcal{A}^2}{c^5 \mu_0}. \quad (11)$$

For generators using petawatt laser pulses of $E_0 \sim 10^{15}$ V/m into intense pulsed magnetic fields of $B_0 \sim 100$ T, one gets that the power loss is of order 10^{-21} W per laser shot. However, due to the very short time scales of the GW emission duration, this might be very hard to detect. For this reason, one may wish to adopt another strategy by using a continuous emission of GW, which can be carried out in a generator constituted of a TEM standing wave plunged into a constant intense magnetic field. By using two spatially separated generators[16], it is possible to create standing GWs in the space between them, as illustrated in Figure 1. From Eq. (9), we can see that the generated GWs have the same frequency as the generating TEM wave and that their amplitude decrease with the inverse of the distance. Two generators with $E_0 = 10^6$ V/m, for in-

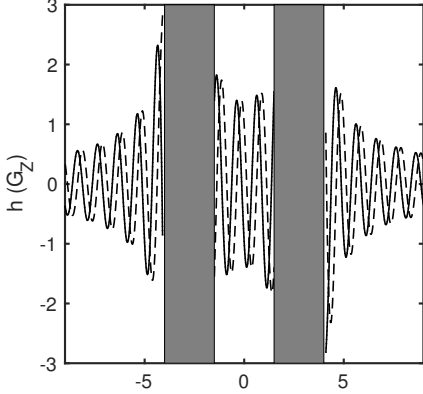


FIG. 1: Metric perturbations h_{00} and h_+ outside of two GW generators (the shaded regions) with $B_{x,y} = \sqrt{2}/2 B_0$ and a circularly polarized TEM wave (such that $h_{00} = h_{12}$) from analytical solutions of Eq.(9)

If we now assume for the detector the same configuration of a TEM wave $\mathcal{E}_{x,y}$ into a transverse magnetic field $\mathcal{B}_{x,y}$ as above, the effective current densities in Eq.(13) are given by

$$\begin{aligned} \mathcal{J}^x &= \partial_z h_{\times} (\mathcal{B}_x + [\mathcal{E}_y^{\leftarrow} - \mathcal{E}_y^{\rightarrow}] / c) - \partial_t h_{\times} (\mathcal{E}_y^{\leftarrow} + \mathcal{E}_y^{\rightarrow}) / c^2 - \partial_z h_{+} (\mathcal{B}_y - [\mathcal{E}_x^{\leftarrow} - \mathcal{E}_x^{\rightarrow}] / c) - \partial_t h_{+} (\mathcal{E}_x^{\leftarrow} + \mathcal{E}_x^{\rightarrow}) / c^2 \\ \mathcal{J}^y &= \partial_z h_{\times} ([\mathcal{E}_x^{\leftarrow} - \mathcal{E}_x^{\rightarrow}] / c - \mathcal{B}_y) - \partial_t h_{\times} (\mathcal{E}_x^{\leftarrow} + \mathcal{E}_x^{\rightarrow}) / c^2 - \partial_z h_{+} (\mathcal{B}_x + [\mathcal{E}_y^{\leftarrow} - \mathcal{E}_y^{\rightarrow}] / c) + \partial_t h_{+} (\mathcal{E}_y^{\leftarrow} + \mathcal{E}_y^{\rightarrow}) / c^2 \end{aligned} \quad (14)$$

with $\mathcal{J}^\nu = \mu_0 j_{\text{eff}}^\nu$.

We will solve Eq.(13) under its wave form, by letting

stance obtained through a microwave cavity of $\mathcal{A} = 1\text{m}^2$, and $B_0 = 10$ T will produce SGWs of amplitude of order 10^{-39} and an energy loss of 10^{-25} J after one year of continuous operation. It is worth noticing that the somewhat similar experimental design of a microwave cavity in a (although longitudinal) magnetic field used in the ADMX experiment searching for axions has a sensitivity of 10^{-24} W over a time integration of 10^3 s [14]. In addition to this energy loss, it is also possible to remotely detect the generated GWs through their interaction with an EM field.

Electromagnetic detectors of GWs. A GW passing through an external static magnetic field will locally modify the volume, which results in a locally varying magnetic flux and the emission of EM waves. This is called the inverse Gertsenshtein effect[17]: the conversion of GWs into EM ones. This is ruled by the second group of covariant Maxwell equations[18] $\nabla_\mu F^{\mu\nu} = 0$ perturbed at first order in the metric:

$$\begin{aligned} \nabla_\mu F^{\mu\nu} &\approx (\eta^{\mu\alpha} \eta^{\nu\beta} - \eta^{\mu\alpha} h^{\nu\beta} - h^{\mu\alpha} \eta^{\nu\beta}) \partial_\mu F_{\alpha\beta} \\ &\quad - \eta^{\nu\beta} \partial_\mu h^{\mu\alpha} F_{\alpha\beta} - \eta^{\mu\alpha} \partial_\mu h^{\beta\nu} F_{\alpha\beta} \\ &\quad + \frac{1}{2} \partial_\mu h_\sigma^\sigma F^{\mu\nu}. \end{aligned} \quad (12)$$

If we now set $F_{\mu\nu} = F_{\mu\nu}^{(0)} + F_{\mu\nu}^{(1)}$ and if we focus solely on the first order correction $F_{\mu\nu}^{(1)}$, the previous equation simply reduces to

$$\partial_\mu F_{(1)}^{\mu\nu} = -\eta^{\mu\alpha} \partial_\mu h^{\beta\nu} F_{\alpha\beta}^{(0)} \equiv -\mu_0 j_{\text{eff}}^\nu \quad (13)$$

neglecting higher order terms, given that $h_{\mu\nu}$ is traceless and satisfies the Lorenz gauge condition, and considering that zeroth order Faraday tensor components verifies Maxwell equation on Minkowski background $\partial_\mu F_{\alpha\beta}^{(0)} = 0$. Equation (13) shows us that the GWs $h_{\mu\nu}$ can combine with background EM fields $F_{(0)}^{\mu\nu}$ to constitute an effective 4-current density j_{eff}^ν that generates weak EM fields $F_{(1)}^{\mu\nu}$.

$F_{\mu\nu}^{(1)} = \partial_\mu A_\nu^{(1)} - \partial_\nu A_\mu^{(1)}$ and assuming Lorenz gauge

$\partial_\mu A_{(1)}^\mu = 0$ such that

$$\square^2 A_\mu^{(1)} = \mu_0 j_\mu^{\text{eff}}. \quad (15)$$

It can also be shown that $j_{\text{eff}}^t = j_{\text{eff}}^z$ so that $A_t^{(1)} = -A_z^{(1)}$ and those degrees of freedom can therefore be gauged away[19].

In order to build a couple of gravitational emitter-receiver, one can consider two types of GW electromagnetic detectors to be associated to the previously introduced generator of SGWs. The first type would be to rely solely on intense petawatt laser pulses propagating into the continuous incoming flux of GWs, so that $\mathcal{E} \sim 10^{15} \text{V/m}$ for very short times and no static magnetic field $\mathcal{B} = 0$. In this case, the SGWs will produce a very short and faint EM pulse whose potential $A_{x,y}^{(1)}$ is of order of magnitude $\mathcal{E}/cL\mathcal{G}_Z$ (where L is the distance the laser pulse covers into the flux of GWs). The amplitude and time scales of the resulting EM waves are that small that one could prefer considering a second type of detector by letting the incoming GWs interacting solely with a static external magnetic field $\mathcal{B} \neq 0$ while $\mathcal{E} = 0$. This configuration allows one to produce EM waves continuously into the detector, so that their accumulation could be detected despite of their faintness at generation.

In order to provide a simple model of this, we will assume a detector made of a one-dimensional EM ideal cavity plunged into an external static magnetic field of amplitude \mathcal{B} . Eq. (15) can now be solved using

$$A_{x,y}^{(1)}(Z, T) = \frac{\mathcal{B}L\mathcal{G}_Z}{2} \int_0^T \int_{Z-(T-T')}^{Z+(T-T')} \mathcal{F}_{x,y}(Z', T') dZ' dT' \quad (16)$$

where $T = ct/L$, $Z = z/L$, $\mathcal{F}_{x,y}$ can be obtained from Eqs.(14) and if we assume $A_{x,y}^{(1)}(Z, 0) = \partial_T A_{x,y}^{(1)}(Z, 0) = 0$. The associated electric and magnetic fields are transverse and respectively given by $|e_{x,y}| = \partial_t A_{x,y}^{(1)}$ and $|b_{x,y}| = \partial_z A_{y,x}^{(1)}$.

An experimental concept for GW emitter-receiver. The ideal experimental set-up we consider is illustrated in Fig. 2: it is made of two separate GW emitters, each consisting of a TEM cavity plunged into an external static magnetic field, surrounding a central GW receiver: a resonant cavity also in a transverse static magnetic field. The emitter on the left (right) will send GWs $h_{+, \times}^{\rightarrow} (h_{+, \times}^{\leftarrow})$ into the magnetic field of the detector, where these perturbations react to regenerate EM waves $A_{x,y}^{(1) \rightarrow, \leftarrow}$. Each of these waves will be reflected on the edges of a central EM cavity so that the total amplitude increases with time.

An example of the EM potentials $A_{x,y}^{(1) \rightarrow, \leftarrow}$ remotely induced into the magnetic field of the detector is illustrated in Fig. 3. Each of the passing forward and backward propagating GW $h_{+, \times}^{\rightarrow, \leftarrow}$ generates a forward/backward

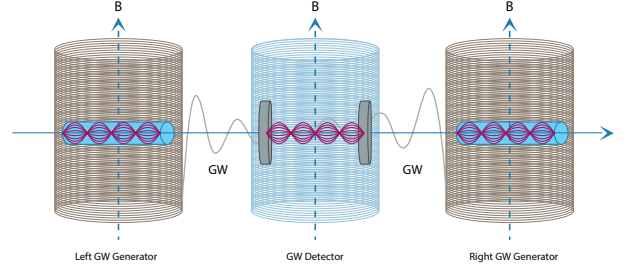


FIG. 2: Experimental concept for GW emitter-receiver, using resonant EM cavities plunged into external transverse magnetic fields

propagating EM wave $A_{x,y}^{(1) \rightarrow, \leftarrow}$ whose amplitude increases along the path of the GW propagation. The combination of both induced EM wave, $A_{x,y}^{(1) \rightarrow} + A_{x,y}^{(1) \leftarrow}$, constitutes a standing wave pattern whose amplitude is extremely faint, of order $\mathcal{B}L\mathcal{G}_Z$.

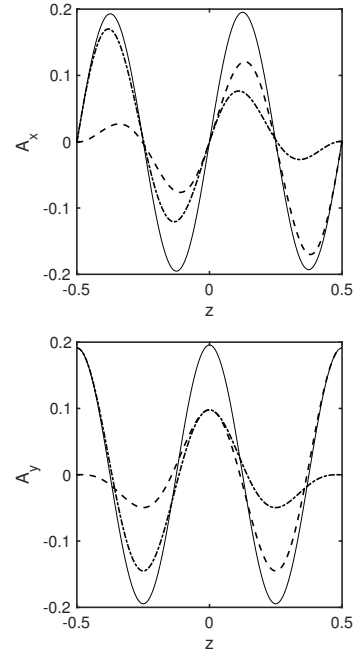


FIG. 3: Remotely induced EM potentials $A_{x,y}^{(1)}$ (in units of $\mathcal{B}L\mathcal{G}_Z$): $A_{x,y}^{(1) \rightarrow}$ (dashed line), generated by $h_{+, \times}^{\rightarrow}$; $A_{x,y}^{(1) \leftarrow}$ (dash-dotted line), due to $h_{+, \times}^{\leftarrow}$ and $A_{x,y}^{(1) \rightarrow} + A_{x,y}^{(1) \leftarrow}$ (thin solid line)

However, it is possible to increase the signal through the use of reflecting edges inside the detector. In an ideal case, each generated wave $A_{x,y}^{(1) \rightarrow, \leftarrow}$ will therefore be kept inside the cavity of the detector and, since they are continuously created by the incoming GWs of the generators, the total amplitude of $A_{x,y}^{(1)}$ will grow up linearly with time. Fig. 4 represents the rise of the energy density $u = (\epsilon_0 |e|^2 + |b|^2/\mu_0)/2$ at the center $z = 0$ of the detector as time evolves, once taken into account the con-

tinuous production of EM waves and their accumulation through the perfect reflexions on the edges. The energy density evolves quadratically with time [20] boosting up the instantaneous production of EM waves inside the detector by a factor $(ct/L)^2$ (see also [5]). However, even in this case, the amount of radiation remotely induced is extremely weak: for $E_0 = 10^6 \text{ V/m}$, $B_0 = \mathcal{B} = 10 \text{ T}$ and $L = 1 \text{ m}$ one gets $u \sim 10^{-70} (ct/L)^2 \text{ J/m}^3$.

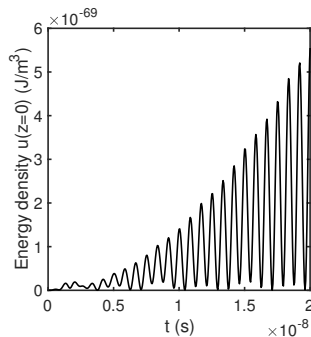


FIG. 4: Time evolution of the energy density $u = (\epsilon_0 |e|^2 + |b|^2 / \mu_0) / 2$ of the EM fields e, b remotely induced in the detector by the SGWs

Conclusion. Controlling gravity, for instance through producing then detecting artificially generated gravitational waves, does not require any new physics nor technology. Indeed, it is in principle achievable within standard general relativity, notably through the Gertsenshtein-Zeldovich effect, and the use of high-field magnets surrounding electromagnetic cavities or high-power lasers. A gravitational counterpart of the Hertz experiment for electromagnetism is conceivable with present technology, although this still requires to identify a very sensitive detection process. The extreme faintness of the gravitational interaction yields that the possibly observable effects include the cumulated energy loss due to GW emission and the energy remotely gained with the EM fields induced by the passing GWs.

We claim that such gravity control experiment will constitute a unique test of the equivalence principle in laboratory, involving relativistic sources in the weak gravitational field limit and in the quantum vacuum of electromagnetism. Harnessing gravity, the last indomitable fundamental interaction, constitutes a true experimental challenge, but will undoubtedly lead to rewarding scientific breakthroughs and new technologies.

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- [15] Following a similar algebraic reasoning, it can be shown that other polarizations (TM/TE) of the EM wave do not generate both h_+ and h_\times , at least as long as a linear geometry is assumed (see also [6]).
- [16] The length of both generators, as well as their spacing, is assumed to be multiple of the TEM wavelength.
- [17] Although unfortunately discarded as being "hardly of interest" by Gerstenshtein himself in his original paper.
- [18] The first group, $\nabla_{[\alpha} F_{\beta\gamma]} = 0$, being unchanged in the Levi-Civita connection.
- [19] In fact, $j_{\text{eff}}^{t,z}$ are sourced by h_{01} and h_{02} which depend only on longitudinal components B_z of the magnetic field. j_{eff}^t and j_{eff}^z are always equal even in the presence of B_z such that $A_0^{(1)}$ and $A_z^{(1)}$ can always be gauged away, leaving only transverse components of the induced EM fields.
- [20] Power losses due to the imperfections of the cavity could also be considered, see for instance [5].